Problem Set 1 for CaSB 186
Due January 20, 2017 in Lab section
(Late problem sets will lose 10 points per day)

1. In class we discussed the equation and solution for logistic growth (Example 2.9 in book) for a population limited by resources, materials, space, competition, etc. Find an approximate or asymptotic solution to the equation and simplify your exact solution in the case that the initial population size is very small compared with the carrying capacity: N(0)<<K, meaning the initial population size is much smaller than the carrying capacity K. Show that you get the same result whether you start from the equation or the exact solution. Which method is easier? Repeat the problem in the case that a population starts growing with an initial population size that is very close to the carrying capacity, N(0)≈K.

2. In a paper in Science in 1960, researchers noted that human population size was growing super exponentially, meaning much faster than exponentially, and that according to this model, we would have infinitely many people by 2026 if growth continued at its current pace.
    a. Approximate super exponential growth rate as the growth rate increasing proportional to N^2 instead of just linearly with N. Write down the differential equation for this. Solve this equation for the time at which the population would become infinitely large.
    b. Can you provide an explanation for why the population growth rate might increase with N^2?
    c. Explain why the population would not become infinitely large.
    d. Since 1960 has the relative or percent population growth rate increased or decreased. Assuming population growth rate is proportional to N^k, how can you estimate the value of k using the following data for Wikipedia.

Population	1	2	3	4	5	6	7
Year	1804	1927	1960	1974	1987	1999	2011
Years elapsed	—	123	33	14	13	12	12

3. Example 2.10 in your book discusses Gompertz growth that has been used to model tumor growth as well as survivorship curves that are characterized by a slowing or decelerating growth rate as the tumor grows larger. As discussed there, this model replaces constant growth rate, r, with a growth rate that decreases in time and thus requires a new additional equation. In your textbook the two equations that define the model are given by $\frac{dN(t)}{dt} = c(t)N(t)$ and $\frac{dc(t)}{dt} = -\propto c(t)$.
    a. Show the solution to these equations is given by $N(t) = N(0)e^{\frac{c_0}{\alpha}(1-e^{-\alpha t})}$. What happens to this solution for long times? Plot the solution versus time by hand or using a computer.
    b. If the sign of $\propto$ is reversed, how do the solution, behavior at long times, and

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plot versus time change?

c. For cases a. and b. here, one case is much more like super-exponential growth in problem 2 and one case is much more like logistic growth in problem 1, which is which? Discuss the advantage and disadvantages of Gompertz growth versus super-exponential growth and versus logistic growth. Which would you choose to use and why? In practice, if you have empirical data, is it possible to distinguish between these different models.

4. Many patterns in nature that arise from some sort of "growth" process that can be mapped onto the Fibonacci sequence. Examples include the nautilus shell, petals of a flower, and leaves on a tree. Fibonnaci derived this rule by thinking about the population growth rate of rabbits. The Golden Ratio, which is related to the Fibonacci sequence (see below), is often used frequently in art.

a. The Fibonacci rule is given by $a_{n+1} = a_n + a_{n-1}$. For developmental or growth processes, n could be thought of as a time step and this rule could be thought of as an update rule. Show that the in the limit n->∞, the ratio of consecutive coefficients is the Golden Ratio $\dfrac{1 \pm \sqrt{5}}{2}$. (Hint: Write the ratio as an update rule and look for fixed points (i.e., convergence)). How many pieces of memory are needed at each step in this process to obtain this ratio? From this standpoint, is the rule's pervasiveness in nature surprising or just one of the simplest things possible?

b. From a more abstract mathematical standpoint, what is the trajectory of the coefficient if the ratio starts out at a_0=-1, -1/2, 1, or 2?

c. If we extend the Fibonacci rule to require one more piece of memory $a_{n+1} = a_n + a_{n-1} + a_{n-2}$, what is the value of the ratio of consecutive coefficients in the limit n->∞?

d. As stated the Fibonacci rule and Golden ratio appear often in nature. Do you think our new rule appears often in nature? Why or why not?

5. In a paper in Science in 1962, researchers wanted to test the effects of the drug LSD on a young male elephant, Tusko, to see if it mimicked the effects of being "on musth", which occurs about once a year in male elephants and corresponds to a period of extreme aggressiveness and irritability. Cats typically received about 0.6 mg of LSD. Elephants weigh roughly 1000 times more than a cat. What factors do you need to consider in order to estimate how much LSD should be given to the elephant? In the study, Tusko died shortly after receiving the dosage.