Problem Set 2 for CaSB 186
Due February 3, 2017 in Lab section
(Late problem sets will lose 10 points per day)

1. Interactions occur among genes, proteins, and species. We can use a common framework of networks and graphs to describe this even if the nodes and edges/links and dynamics along those links are quite different.
   a. If there are 4 genes, proteins, species, drugs, etc. that are labeled W, X, Y, and Z, draw all disconnected graphs that do not involve the drug/gene/protein/species Z. That is, draw the graphs for all interactions that do not involve the Z, including higher-order emergent ones.
   b. Now determine how often similar graphs will show up when drugs W, X, and Y are each excluded, and use that to calculate the total number of disconnected graphs for interactions among 4 objects. Are there any of these that you would not need to subtract off from the net interaction if we want to calculate a 4-way emergent interaction?

2. a. For gene epistasis networks, the metric is usually growth rate of the organism. How does this map onto our equations from class for three-way interactions?
   b. Consider a protein-protein interaction network with measures for binding affinity or protein binding rate? What would our equations for the three-way interaction metrics DA and E3 look like? In relating to our system in class, would there be analogues in this system to drugs and/or bacteria (e.g., E. Coli) and its growth rate.
   c. For a predator-prey system, the predators are often treated analogously to drugs (some stressor) from class and the prey as similar to the bacteria (E. Coli). From this perspective, would a system with two predators that feed on the same prey be a two-way or a three-way interaction? What would the interaction be between?
   d. For a system with two predators and one prey, would this represent a two-way or three-way interaction? If $s_{XY}=0.6$, $s_X=0.8$, and $s_Y=0.8$, does this represent an additive, antagonistic, or synergistic interaction? Calculate this both with and without rescaling. Does the rescaling change your conclusions if the cutoff for antagonism requires the rescaled metric to have a value greater than 0.5 and the cutoff for synergy is the rescaled metric being less than -0.5? What if $s_Y=0.6$ instead?
   e. For three predators and one prey, are there any emergent interactions when $s_{XYZ}=0.55$, $s_{XY}=0.64$, $s_{YZ}=0.64$, $s_{XZ}=0.64$, $s_X=0.8$, $s_Y=0.8$, and $s_X=0.8$? What if $s_{XYZ}=0.30$? For this case, do not worry about any rescaling.

3. a. Consider the network growth model of preferential attachment, but now add two new vertices per time step to the network from the previous time step. Each new vertex still gets attached to m other vertices at the time step it is first introduced. Does this change in the rules affect the exponent for how the connectivity, k_i, of vertex, i, depends on time t? If so, how?
   b. Repeat problem 3, but now let the number of new vertices introduced at time t scale like a power law, as $V(t)=V_0t^p$. How does this affect the scaling of

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the connectivity, k_i, of vertex, i, with time t? For any sums over powers of t, you can approximate those as the corresponding integrals over powers of t. Can the relationship between k_i and t still be expressed as time t raised to an exponent (i.e., a power law)? If not, what form does it take? What is its behavior at long times as t approaches infinity?
(Hint: Be careful when computing the *total* number of edges at time t.)